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Theorem 8.1, namely the strong maximum principle for extended quasilinear differential inequalities of the form (8.2), is in general not correct unless additional conditions are placed on the matrix $[a_{ij}]$. This is because the assertion on page 42, that the product matrix $[a_{ik}b_{kj}]$ is positive definite, fails to hold for arbitrary positive definite matrices $[a_{ij}]$. That is, the (symmetrized) product of two positive definite matrices need not be positive definite; for example, for the matrices

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} 14 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{this product is} \qquad \frac{1}{2} \begin{bmatrix} 56 & 15 \\ 15 & 4 \end{bmatrix},$$

whose determinant is $-1/4$.

A sufficient though rather special condition under which Theorem 8.1 remains valid is (here and in what follows we use the notation of the original paper)

$$(i) \qquad\qquad a_{ij}(x,u) = a(x,u)\delta_{ij},$$

with $a : \Omega \times \mathbb{R}_0^+ \to \mathbb{R}^+$ of class C^1.

Suppose further concerning the differential inequality (8.2) that

$$(1) \qquad\qquad \lim_{\varrho \downarrow 0} \frac{\varrho A'(\varrho)}{A(\varrho)} = c, \qquad c > -1.$$

Then a more general sufficient condition for Theorem 8.1 to be valid is that either $c = 0$ or $c \neq 0$ and the positive definite matrix $[a_{ij}]$ satisfies also

$$(ii) \qquad\qquad \sup_{x \in \Omega} \sqrt{\frac{\Lambda(x)}{\lambda(x)}} < \frac{2 + c + 2\sqrt{1+c}}{|c|},$$

where

$$\Lambda(x) = \max\{\text{eigenvalues of } [a_{ij}(x,0)]\}, \qquad \lambda(x) = \min\{\text{eigenvalues of } [a_{ij}(x,0)]\}.$$

Condition (i) is the general case when the second order part of (8.2) has the variational form

$$(2) \qquad\qquad \text{div}\{a(x,u)A(|Du|)Du\}.$$

Condition (ii) applies to the p–Laplace operator $A(\varrho) = \varrho^{p-2}$, $p > 1$, with $c = p - 2$. Moreover if $c = 0$ in (1), as occurs for example when $A(\varrho) = 1/\sqrt{1 + \varrho^2}$, i.e. the mean curvature operator, then Theorem 8.1 is correct even with no additional conditions on $[a_{ij}]$ outside of positive definiteness and regularity.

The validity of Theorem 8.1 can also be asserted *if the differential inequality (8.2) is assumed to be elliptic for all arguments $(x, 0, Du)$, with $(x, Du) \in \Omega \times \mathbb{R}^n$ such that $0 < |Du| < b$, for some $b > 0$.*

For Theorem 8.5, namely the compact support principle for extended inequalities (8.10), the necessity part is valid as stated. On the other hand, the proof of sufficiency relies on

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Theorem 8.1, so that this part of Theorem 8.5 requires one of the additional conditions (i) or (ii) given above.

Similar remarks apply for Corollaries 8.3, 8.4 and 8.6, as well as for Theorems 9.1 and 9.2. The important Theorem 9.3, however, is correct as stated.

Condition (ii) is based on a result of Nicholson (Linear Algebra and it Applications, vol. 24, 1979, in particular Theorem 2 on page 181, which gives a sufficient condition for the symmetrized product of two positive definite Hermitian matrices to be positive definite).

We wish to thank Professor Charles H. Conley for his valuable insight and help with the present problem. Proofs and further discussion will appear in the forthcoming paper *"Elliptic equations and products of positive definite matrices"*, by C.H. Conley, P. Pucci and J. Serrin.

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